Lab course: Optimization with PDE constraints

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April 2014

Practical example: simulation of a chemical combustion

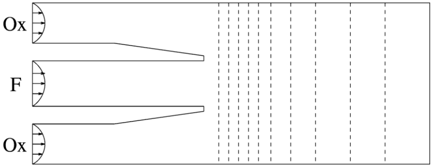


Figure: configuration of a reaction chamber

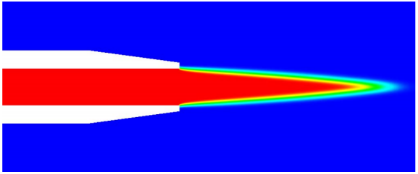


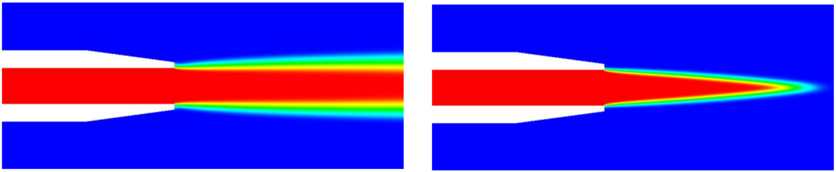
Figure: finite element simulation of the resulting ‘‘flame’’

Modeling the combustion problem

- based on the underlying reaction diffusion equation

$$\beta \cdot \nabla u - \nabla \cdot (\sigma \nabla u) + A \exp\left(-\frac{E}{d-u}\right) u(c-u) = 0 \quad \text{in } \Omega$$
$$u = \hat{u} \quad \text{on } \Gamma_{\text{in}}$$
$$\partial_n u = 0 \quad \text{on } \partial\Omega \setminus \Gamma_{\text{in}}$$

- the parameters (A, E) are empirical and not known a priori



(a) $A = 54.5, E = 0.15 \rightsquigarrow$ incorrect simulation

(b) correct (A, E)?

Parameter identification

- use measurements obtained in real life simulations
- fit the data to the measurements

$$\text{Minimize } J(u) = \frac{1}{2}\|C(u) - C_{meas}\|_Z^2$$

with $u \in \hat{u} + V$ under the constraints

$$(\beta \cdot \nabla u, \varphi) + (\sigma \nabla u, \nabla \varphi) + (s(u,q), \varphi) = 0 \quad \text{for all } \varphi \in V$$

- the control $q = (A, E)$ enters in

$$s(u,q) = s(u,(A,E)) = A \; exp\left(-\frac{E}{d-u}\right) u(c-u)$$

Parameter identification

- use measurements obtained in real life simulations
- fit the data to the measurements

$$\text{Minimize } J(u_h) = \frac{1}{2}\|C(u_h) - C_{meas}\|_Z^2$$

with $u_h \in \hat{u}_h + V_h$ under the constraints

$$(\beta \cdot \nabla u_h, \varphi_h) + (\sigma \nabla u_h, \nabla \varphi_h) + (s(u_h, q), \varphi_h) = 0 \quad \text{for all } \varphi_h \in V_h$$

- the control $q = (A, E)$ enters in

$$s(u, q) = s(u, (A, E)) = A\, exp\left(-\frac{E}{d-u}\right) u(c-u)$$

Objectives of the lab course

What are the objectives ?

- put into practice the lecture "Optimization with PDE constraints"

What will you do ?

- discretize elliptic optimal control problems in MATLAB
  - discretize a forward PDE problem with FEM (cf. lecture previous semester)
  - work with a discrete Lagrangian
  - find the discrete adjoint equation
  - deduce discrete optimality conditions
- compute optimal controls with first and second order methods
- try it on some practical examples

Practical details

on the format

- period : end of semester, lecture free period.
- organization : 4 days in a row, 2 hours tutorials, 4 hours programming exercise
- good preparation for master thesis, etc. (no exam)

required skills

- lecture OptPDE
- MATLAB (familiarity with FE is a plus)

under construction

- room/precise date : to be announced
- registration : at the end of the semester