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Apr 25, 2015

Exercise 2 - Numerical methods for fluid-structure interaction (Summer term 2015)

Exercise 2.1: Let ρ, v, f, σ be sufficiently smooth. Furthermore, let x be the position of a rotating mass point with origin x_0 and it holds the conservation of angular momentum, i.e.,

$$\int_\Omega (x - x^0) \times \Big(\partial_t(\rho v) + \sum_{j=1}^{3} \partial_{x_j}(\rho v_j v) \Big)\, dx = \int_\Omega (x - x^0) \times (\rho f)\, dx + \int_{\partial\Omega} (x - x^0) \times (\sigma n)\, ds.$$

Show that σ is symmetric, i.e.,

$$\sigma = \sigma^T.$$

Hints: First, the operation $\times$ denotes the cross product. As second hint, multiply both sides of the above formula with $a \in \mathbb{R}^3$ and set them at the end $a = e_1, a = e_2, a = e_3$ in order to obtain the symmetry component wise.

Exercise 2.2: Rivisiting Reynolds theorem.

- Recapitulate why $\hat{J} > 0$ is a necessary condition in Reynold's theorem.
- Either give precise arguments or a mathematical proof that Reynolds' transport theorem does not have convective terms when applied in the reference configuration $\hat{\Omega}$, i.e.,

$$D_t \int_{\hat{\Omega}} \hat{T}(t, \hat{x})\, d\hat{x} = \int_{\hat{\Omega}} \partial_t \hat{T}(t, \hat{x})\, d\hat{x}.$$

Exercise 2.3: Provide either physical descriptions or mathematical definitions of the following properties:

- Rigid body motion;
- Homogeneous material;
- Isotropic elastic materials;
- Material frame-indifference (any quantity must be independent of the particular orthogonal basis in which it is computed)

Hint: This last exercise requires a bit of literature research.

Discussion of exercises: May 4, 2015