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Jun 11, 2015

Exercise 5 - Numerical methods for fluid-structure interaction (Summer term 2015)

Exercise 5.1

Show that the divergence-free condition $\nabla \cdot v = 0$ reads in ALE$_{fx}$ formulation

$$\hat{\nabla} \cdot (\hat{J}\hat{F}^{-1}\hat{v}) = 0.$$

Exercise 5.2:

- Recapitulate and explain A-stability for ordinary differential equations (ODEs).
- Why are such schemes preferable for temporal discretization of partial differential equations?

Exercise 5.3:

Show that applying the One-Step-θ scheme to the strong form (e.g., the heat equation) and then deriving the weak formulation yields the same result as first deriving the weak form and then applying the One-Step-θ scheme.

Exercise 5.4:

Apply the BDF(2) time-stepping formula

$$\delta^{-1}(\frac{3}{2}y_n - 2y_{n-1} + \frac{1}{2}y_{n-2}) = f(t_n, y_n),$$

to the Navier-Stokes equations. Here, y_n is the unknown solution at time t^n and δt is the time step size. (Remark: It is sufficient to consider the Navier-Stokes equations in their natural Eulerian framework.)

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Exercise 5.5:

Apply the One-Step-θ scheme to the 2nd order-in-time elasticity equation,

$$\hat{\rho}_s \partial_t \hat{u}_s - \hat{\nabla} \cdot (\hat{F}\hat{\Sigma}) = \hat{\rho}_s \hat{f}_s. \tag{0.0.1}$$

Write this equation into first order-in-time mixed form before you apply the One-Step-θ scheme.

- Set $\theta = 1$ and show that the second variable, the velocity $\hat{v}_s$, can be replaced such that the resulting scheme does only depend on the displacements $\hat{u}_s$. (Hint: the result is the same as if we would have applied directly a second-order central difference quotient to Equation (0.0.1).
- Does this derivation also hold for $\theta = 0.5$?

Discussion of exercises: Jun 15, 2015