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Exercise 6 - Numerical methods for fluid-structure interaction (Summer term 2015)

Exercise 6.1

Let the following nonlinear initial/boundary-value problem be given: Find vector-valued displacements $u : \mathbb{R}^3 \to \mathbb{R}^3$ for all times $t \in [0, T]$ such that

$$\partial_t u - \nabla \cdot (F\Sigma) = f \quad \text{in } [0,T] \times \Omega, \tag{0.0.1}$$

$$u = 0 \quad \text{on } [0,T] \times \partial\Omega, \tag{0.0.2}$$

$$u = 0 \quad \text{in } \{t = 0\} \times \Omega. \tag{0.0.3}$$

Here, $F = I + \nabla u$ and $\Sigma = \mu \nabla u$ where I is the identity matrix and $\mu > 0$.

1. Derive the time-discretized scheme using a backward Euler discretization;
2. Derive the weak formulation on the spatially continuous level;
3. Next, apply a Galerkin FEM scheme (e.g., using Q_1^c elements) for spatial discretization;
4. Apply (formally) Newton's method to linearize the problem;
5. Write down the linear equation system that needs to be solved in each Newton step.

Discussion of exercises: Jul 6, 2015