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Summary - Numerical methods for fluid-structure interaction (Summer term 2015)

Questions, tasks, keywords

1. How did we define FSI in this lecture? (Hint: which physical equations are coupled?)
2. What are the underlying physical principles and equations to study FSI?
3. Please write down the equations for conservation of mass and momentum.
4. What is the mathematical characterization of these underlying equations? (Hint: What type of PDEs are we dealing with?)
5. What is one of the typical difficulties of FSI? (Hint: Eulerian and Lagrangian coordinates)
6. Could you name a second challenge of FSI? (Hint: either as we had in the class or your own opinion)
7. What are the key quantities in continuum mechanics?
8. What is the physical interpretation of $\hat{J} > 0$?
9. What does mathematically happen if $\hat{J} \to 0$?
10. Why do we need the Piola transformation?
11. Write down the Green-Lagrange strain tensor $\hat{E}$.
12. Why do we need constitutive laws? Do you have an example for a fluid or a solid?
13. Which law from continuum mechanics yields the symmetry of Cauchy's stress tensor?
14. What is the difference between the 1st and 2nd Piola-Kirchhoff stress tensor?
15. How are incompressible fluids and solids characterized?
16. Write down the isothermal, incompressible Navier-Stokes equations in their natural framework.
17. What does the Reynolds number characterize?
18. How did we proof existence of linearized elasticity? Which assumptions need to be checked therein?

19. Write down the weak formulation of Navier-Stokes in natural coordinates.

20. What is the purpose of the inf-sup condition (on the continuous level) for Navier-Stokes?

21. What are the physical coupling conditions of fluid-structure interaction?

22. What is the difference between monolithic and partitioned coupling and their classification within strongly and loosely-coupled approaches?

23. What is the idea of ALE (arbitrary Lagrangian-Eulerian)?

24. Define the ALE transformation and the ALE time derivative. (Hint: either in words or in terms of mathematical formulae).

25. Using the ALE approach, which equation needs to be transformed to the reference configuration?

26. Explain variational-monolithic coupling.

27. What is the difference between ALE_{fx} and ALE_{dm}?

28. What is the 'third' condition using an ALE approach for FSI?

29. What is the purpose of MMPDEs?

30. How can we improve mesh regularity in an ALE approach? How is this realized in practice?

31. How many equations must be finally solved using an ALE-FSI approach?

32. What are the advantages and drawbacks of ALE-FSI?

33. How did we discretize FSI in this lecture?

34. Time-discretize the Navier-Stokes equations with a One-step-θ scheme.

35. Why do we use a mixed first-order-in-time system for the solid equations?

36. Which numerical stability concept should (must) be used for temporal discretization of fluid-structure interaction?

37. How can we stabilize the Crank-Nicolson scheme? Why is this (sometimes) important?

38. What is the idea of Galerkin FEM?

39. What is the discrete analogon of the inf-sup condition and why is this important?

40. What are the consequences of the discrete inf-sup condition (the BB-condition) for the choice of FEM spaces for spatial discretization of the Navier-Stokes equations?

41. Describe or just state the Taylor-Hood element.

42. What are numerical issues of the solution of Navier-Stokes for high Reynolds numbers?

43. Write down (without any hats) Newtons's defect correction for: Find $U_h \in X_h$ such that
$$A(U_h)(\Phi_h) = F(\Phi_h) \quad \forall \Phi \in X_h$$

44. Apply Newton's method to
$$A(u)(\varphi) = (u \cdot \nabla u, \varphi).$$

45. How do we determine the block structure of a linear equation system of a coupled PDE system? What is the block structure of the incompressible Navier-Stokes equations?

Discussion planned for Jul 8, 2015