Exercises for Advanced Topics in High Performance Scientific Computing
WS 15/16, Nr.: 0000003213, 02.10.011, Rechnerraum
Exercise Sheet 10 (until December 23, 2015)

Exercise 10*
Use the finite difference method to discretize the Laplace equation on the unit square

$$\Delta u(x,y) = 0, \quad (x,y) \in \Omega := (0,1) \times (0,1) \tag{1}$$

with the boundary conditions given by

$$u(x,0) = u(x,1) = 0, \quad u(0,y) = u(1,y) = \sin(\pi y), \quad x, y \in [0,1] \ . \tag{2}$$

Solve the resulting linear system $Au = f$ with the Jacobi method with an initial guess $u^{(0)} = u_0$ and for $k > 0$ calculate

$$u_i^{(k+1)} := \frac{1}{a_{ii}} \left(f_i - \sum_{\substack{j=1 \\ j \neq i}}^{n} a_{ij} u_j^{(k)} \right), \ i = 1, \ldots, n \ . \tag{3}$$

Implement the Jacobi iteration in C/C++ with a matrix-free approach and parallelize the matrix evaluation using CUDA. Assign every degree of freedom in equation (3) a single CUDA thread in a 1D or 2D thread block and grid layout and use a hierarchical reduction to calculate the error norm within the CUDA kernel. Terminate the Jacobi iteration if the error norm $\|f - Au\|_2 < 10^{-6}$. Analyze the parallel efficiency of the Jacobi solver in a graph for different grid sizes and number of threads.

* Place all source files of the exercises in a folder named `Exercise10` in your home directory on the `mephisto.uni-graz.at` cluster.