Exercises for Advanced Topics in High Performance Scientific Computing
WS 15/16, Nr.: 0000003213, 02.10.011, Rechnerraum
Exercise Sheet 4 (until November 11, 2015)

Exercise 4* Define the open ball $B_p(x_0, r) := \{x \in \mathbb{R}^2 : \|x - x_0\|_p < r\}$. The domain Ω is given as the set complement

$$\Omega = B_\infty((0,0),1) \setminus \overline{B_2((1,-1),1) \cup B_2((0.5,0.5),0.25) \cup B_2((-0.5,-0.5),0.25)} \tag{1}$$

and the boundaries are

$$\Gamma^+ = \partial B_2((0.5,0.5),0.25) \tag{2}$$
$$\Gamma^- = \partial B_2((-0.5,-0.5),0.25) \tag{3}$$
$$\Gamma^0 = \partial(B_\infty((0,0),1) \setminus \overline{B_2((1,-1),1)}) \, . \tag{4}$$

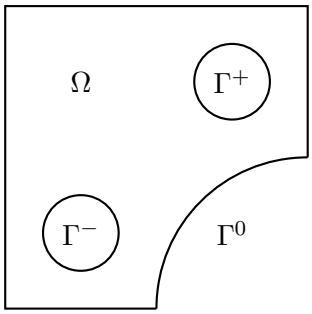

Figure 1: Domain Ω with boundaries Γ^+, Γ^-, and Γ^0.

Solve the heat equation

$$\frac{\partial}{\partial t} u(x,y,t) = \epsilon \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) u(x,y,t), \quad (x,y,t) \in \Omega \times (0,T) \tag{5}$$

with the initial condition

$$u(x,y,0) = 0, \quad (x,y) \in \Omega \tag{6}$$

and boundary conditions

$$u(x,y,t) = \begin{cases} +1, & (x,y,t) \in \Gamma^+ \times [0,T] \\ -1, & (x,y,t) \in \Gamma^- \times [0,T] \\ 0, & (x,y,t) \in \Gamma^0 \times [0,T] \end{cases} \tag{7}$$

and thermal diffusivity $\epsilon = \frac{2}{3}$.
Use the finite difference method to discretize the Laplace operator and the forward Euler time stepping scheme with the condition $4\epsilon\delta t \leq (\delta x)^2$ to calculate the time evolution.

$$u(x_i, y_j, t_{k+1}) = u(x_i, y_j, t_k) + \frac{\delta t}{(\delta x)^2} \epsilon(\Delta u)(x_i, y_j, t_k), \tag{8}$$
$$(x_i = i\delta x, y_j = j\delta x, t_k = k\delta t) \in \overline{\Omega} \times [0,T] \tag{9}$$

Consider how to efficiently implement the finite difference method for the domain Ω!
Visualize the time evolution with Matlab. Export the function $u(x,y,t)$ to Matlab by using the `fstream` class in the C++ template library to write binary data files.

* Place all source files of the exercises in a folder named `Exercise4` in your home directory on the `mephisto.uni-graz.at` cluster.