

Exercises for Advanced Topics in High Performance Scientific Computing

WS 15/16, Nr.: 0000003213, 02.10.011, Rechnerraum

Exercise Sheet 6 (until November 25, 2015)

Exercise 6*

Use the finite difference method to discretize the Laplace equation on the unit square

$$\Delta u(x, y) = 0, \quad (x, y) \in \Omega := (0, 1) \times (0, 1) \quad (1)$$

with the boundary conditions given by

$$u(x, 0) = u(x, 1) = 0, \quad u(0, y) = u(1, y) = \sin(\pi y), \quad x, y \in [0, 1]. \quad (2)$$

Solve the resulting linear system $Au = f$ with the Jacobi method with an initial guess $u^{(0)} = u_0$ and for $k > 0$ calculate

$$u_i^{(k+1)} := \frac{1}{a_{ii}} \left(f_i - \sum_{\substack{j=1 \\ j \neq i}}^n a_{ij} u_j^{(k)} \right), \quad i = 1, \dots, n. \quad (3)$$

Implement the Jacobi iteration in C/C++ with a matrix-free approach. Split the unit square Ω into horizontal stripes Ω_p , $0 \leq p < P$ and parallelize the Jacobi iteration using MPI. Depending on the number of MPI processes P the horizontal stripes are defined as

$$\Omega_p := (0, 1) \times (p/P, (p+1)/P), \quad 0 \leq p < P \quad (4)$$

and the boundaries between the stripes are

$$\Gamma_{p,q} := \overline{\Omega}_p \cap \overline{\Omega}_q, \quad 0 \leq p, q < P. \quad (5)$$

Use the `MPI_Send` and `MPI_Recv` functions to exchange the boundary data between the processes and `MPI_Allreduce` to accumulate the error norm. Terminate the Jacobi iteration if the error norm $\|f - Au\|_2 < 10^{-6}$. Analyze the parallel efficiency of the Jacobi solver in a graph for different grid sizes and number of processes.

* Place all source files of the exercises in a folder named `Exercise6` in your home directory on the `mephisto.uni-graz.at` cluster.