Exercises for Advanced Topics in High Performance Scientific Computing
WS 15/16, Nr.: 0000003213, 02.10.011, Rechnerraum
Exercise Sheet 9 (until December 16, 2015)

Exercise 9*
Use the finite difference method to discretize the Laplace equation on the unit square

$$\Delta u(x,y) = 0, \quad (x,y) \in \Omega := (0,1) \times (0,1) \tag{1}$$

with the boundary conditions given by

$$u(x,0) = u(x,1) = 0, \quad u(0,y) = u(1,y) = \sin(\pi y), \quad x,y \in [0,1] \; . \tag{2}$$

Solve the resulting linear system $Au = f$ with the Jacobi method with an initial guess $u^{(0)} = u_0$ and for $k > 0$ calculate

$$u_i^{(k+1)} := \frac{1}{a_{ii}} \left(f_i - \sum_{\substack{j=1 \\ j \neq i}}^{n} a_{ij} u_j^{(k)} \right), \; i = 1, \ldots, n \; . \tag{3}$$

Implement the Jacobi iteration in C/C++ with a matrix-free approach. Split the unit square Ω into horizontal stripes Ω_p, $0 \leq p < P$ and parallelize the Jacobi iteration using OpenMP. Depending on the number of OpenMP threads P the horizontal stripes are defined as

$$\Omega_p := (0,1) \times (p/P, (p+1)/P), \quad 0 \leq p < P \tag{4}$$

and the boundaries between the stripes are

$$\Gamma_{p,q} := \overline{\Omega}_p \cap \overline{\Omega}_q, \quad 0 \leq p, q < P \; . \tag{5}$$

Use the OpenMP parallel or loop construct to implement the Jacobi iteration and the OpenMP reduction clause or other synchronization primitives to accumulate the error norm. Terminate the Jacobi iteration if the error norm $\|f - Au\|_2 < 10^{-6}$. Analyze the parallel efficiency of the Jacobi solver in a graph for different grid sizes and number of threads.

* Place all source files of the exercises in a folder named `Exercise9` in your home directory on the `mephisto.uni-graz.at` cluster.