Algebraic Multigrid Methods on GPU-Accelerated Hybrid Architectures

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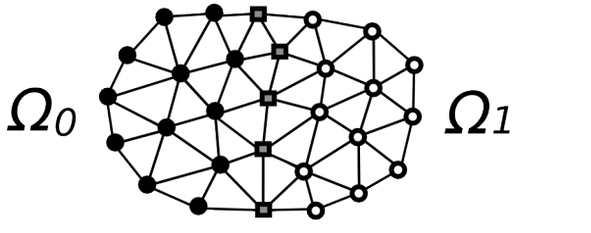
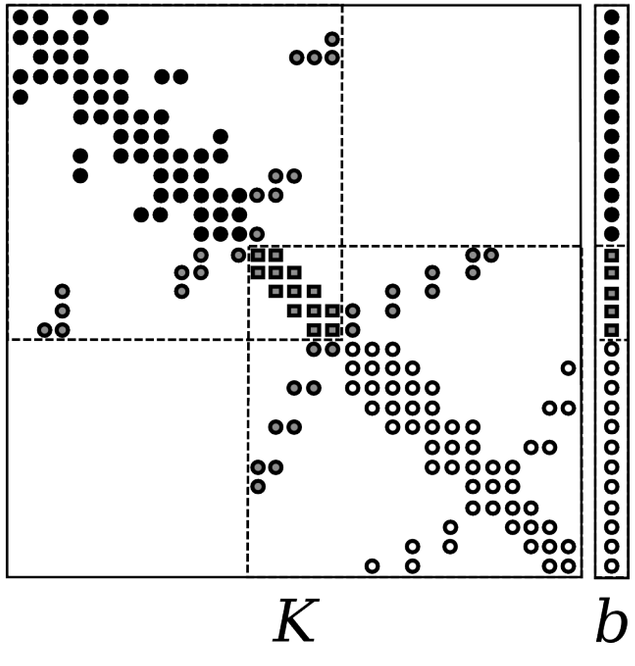
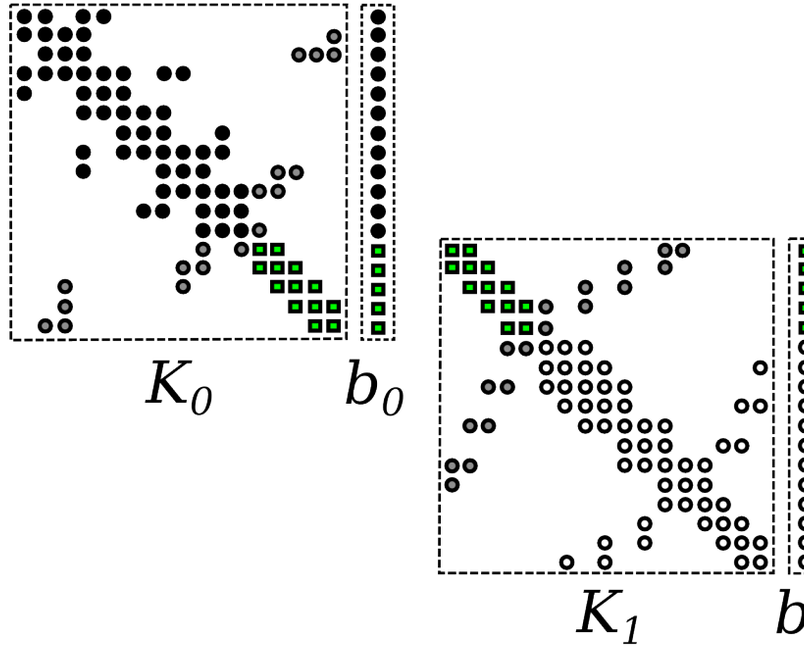
(7) Domain Decomposition Revisited







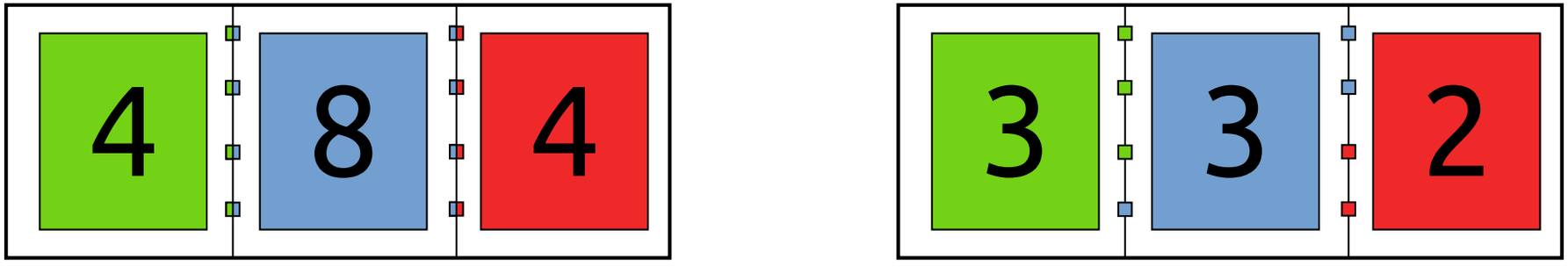
Parallel Scalability Challanges: Data Accumulation

Parallel linear algebra requires an accumulation of the data values on the boundary nodes.

What is holding us back?

- Unbalanced domain decomposition w.r.t. the boundary nodes.
- METIS balances the number of elements per process, i.e. computation
- Up to 100% variation in the boundary sizes, i.e. communication

Performance Tuning: New Data Accumulation Scheme



Assign to every shared boundary node a unique master process. The master process is then responsible for the data accumulation of the boundary node. Use an optimization algorithm to evenly distribute the master processes.

New Data Accumulation Scheme Advantages

The communication complexity is reduced to $\mathcal{O}(2(r-1))$ with r processors participating, instead of $\mathcal{O}(r(r-1))$ for the standard approach, leading to a balanced data accumulation.

Standard Accumulation (STD)

MPI process p executes:

$\mathrm{CopyToBuffer}(f_p, B_{p,q}, \tilde{f}_p)$ {Fill the communication buffer}

$\mathrm{MPIAlltoall}(\tilde{f}_p, B_{p,q}, \tilde{g}_p, B_{p,q})$ {Send data to all neighbors}

$\mathrm{AccumulateInPlace}(\tilde{g}_p, B_{p,q}, f_p)$ {Accumulate in place}

Optimized Accumulation (OPT)

MPI process p executes:

$\mathrm{MPIAlltoall}(f_p, S_{p,q}, \tilde{g}_p, R_{p,q})$ {Send data to masters}

$\mathrm{Accumulate}(\tilde{g}_p, R_{p,q})$ {Accumulation on master processes}

$\mathrm{MPIAlltoall}(\tilde{g}_p, R_{p,q}, f_p, S_{p,q})$ {Receive data from masters}

CPU Execution Times for Accumulation Algorithm

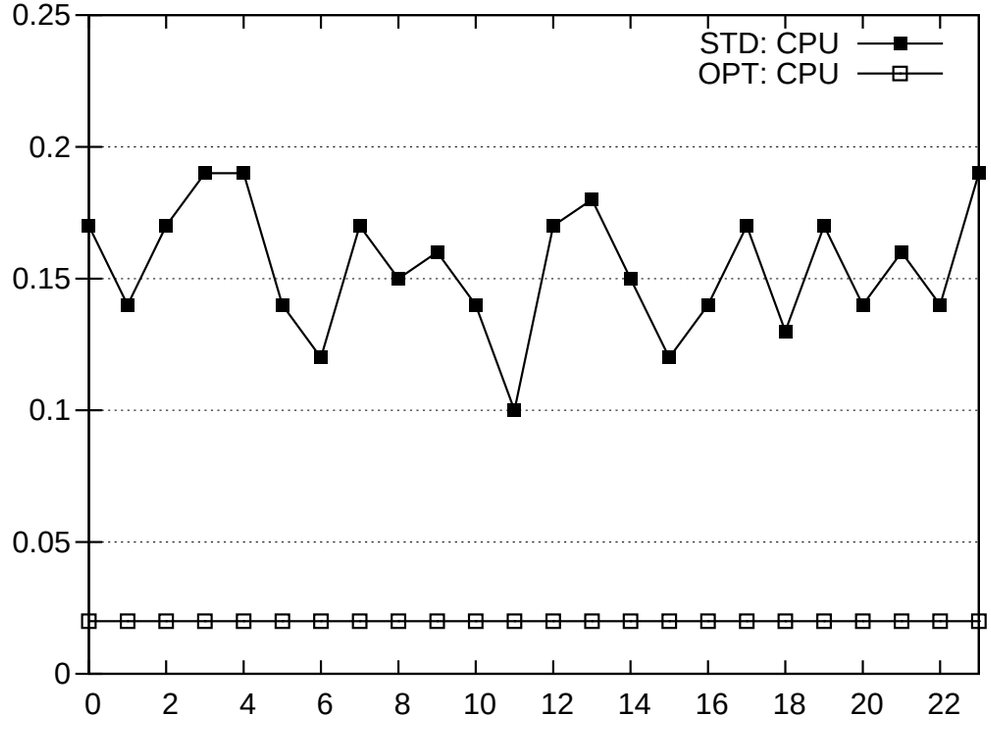


Figure 1: CPU execution times excluding MPI communication times in seconds for the standard (STD) and the optimized (OPT) accumulation algorithm for 24 MPI processes for the benchmark example measured with Scalasca.

MPI Communication Times for Data Exchange

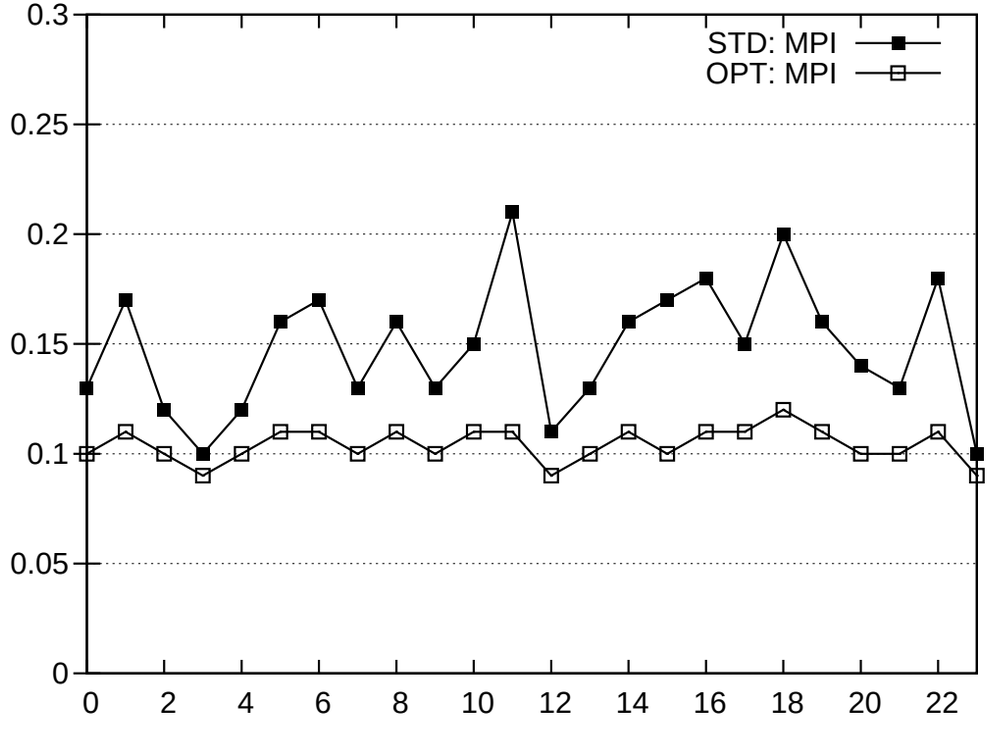


Figure 2: MPI communication times for the standard (STD) and the optimized (OPT) accumulation algorithm for 24 MPI processes for the benchmark example measured with Scalasca.

Total Runtimes for the Full Accumulation Algorithm

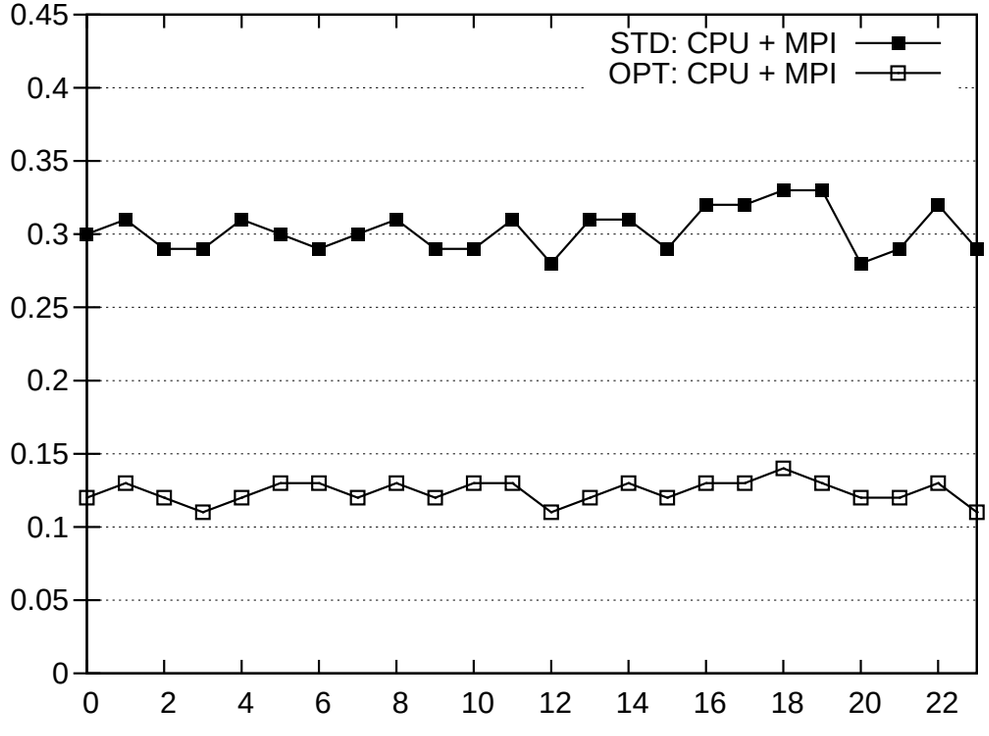


Figure 3: Total runtimes in seconds for the standard (STD) and the optimized (OPT) accumulation algorithm for 24 MPI processes for the benchmark example measured with Scalasca.

Optimization Algorithm

Minimize the function

$$J = \sum_{p=1}^{P} J^p, \quad J^p = \sum_{q=1}^{P} \left(n_q^p - \bar{n}_q^p \right)^2, \quad p \in I \tag{1}$$

w.r.t. the given approximate uniform distribution of the master nodes.

$$\bar{n}_q^p := \left\lfloor \frac{\nu_q^p N^p + N^p}{P} \right\rfloor - \left\lfloor \frac{\nu_q^p N^p}{P} \right\rfloor, \quad p, q \in I \tag{2}$$

with the shift function defined as $\nu_q^p := q + p - 2 \mod P, \quad p, q \in I$ and $N^p := \left| \hat{B}_p \right|$ the size of the restricted boundary node set $\hat{B}_p := \{ v \in B : p - 1 \equiv v - 1 \mod P \}$.

Optimization Algorithm: Implementation

Parallel efficiency is of concern!

- Strategy: Minimize $J^p = \sum_{q=1}^{P} \left(n_q^p - \bar{n}_q^p\right)^2, \quad p \in I.$
- Solve the local optimization problems independently in parallel!
- Randomized local search strategy inspired by the simulated annealing approach.
- Stronger requirement than necessary for the solution of the global optimization problem.
- Advantage: Fully parallel algorithm with no communication during the local optimization.
- Communication: Exchange the master node configuration between the processes.

Optimization Algorithm: Results for Multigrid Hierarchy

Level	Processors 6	12	24	48
1	29461	44581	62756	83738
2	14003	20948	29225	38326
3	4721	6874	9305	11609
4	1285	1793	2180	2413
5	306	347	358	360
6	60	61	65	58
7	15	13	14	12
8	5	5	4	5
9	2	1	2	2
10	1		1	1

Table 1: The global number of boundary vertices for different levels of the AMG algorithm and different numbers of processors.

Optimization Algorithm: Results for Multigrid Hierarchy

	Processors			
Level	6	12	24	48
1	0	0	4	146
2	0	0	10	168
3	0	2	14	372
4	0	12	70	420
5	0	12	88	262
6	0	4	38	82
7	0	2	16	18
8	0	0	6	8
9	0	0	2	4
10	0		2	2

Table 2: Computed values of the functional J for different levels of the AMG algorithm and different numbers of processors.

MPI Communication Times for PCG-AMG Benchmark

PCG-AMG Benchmark: The tetrahedral mesh of a simplified model of a rabbit heart, with 5,082,272 elements and 862,515 vertices.

	OPT			STD		
Processors	min	mean	max	min	mean	max
6	0.03	0.05	0.06	0.02	0.07	0.08
12	0.04	0.05	0.06	0.03	0.06	0.14
24	0.09	0.11	0.12	0.10	0.15	0.21
48	0.27	0.27	0.28	0.52	0.53	0.58

Table 3: MPI communication times in seconds for the standard (STD) and optimized (OPT) accumulation algorithms measured with Scalasca.

PCG-AMG Solver Performance

Parallel efficiency of the PCG-AMG solver is significantly increased.

Processors	Runtime OPT	Runtime STD	Efficiency OPT	Efficiency STD
6	2.38	2.40	1.00	1.00
12	1.22	1.24	0.97	0.97
24	0.69	0.77	0.86	0.78
48	0.57	0.84	0.52	0.36

Table 4: Total runtimes in seconds and parallel efficiency for the PCG-AMG solver for two linear solves.

Mephisto cluster: Five compute nodes with two six-core Intel Xeon X5650 CPUs running at 2.67 GHz and 96 GB RAM per node and a Mellanox 40 Gb/s QDR-Infiniband network.